Name: ______________________________

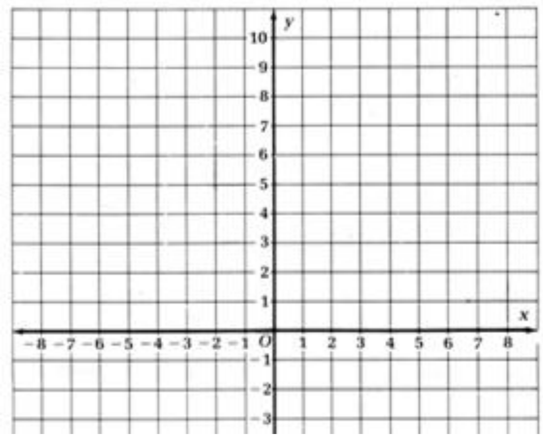
4.1 What are Congruent Figures?

Objectives: Understand the concept of congruent figures
Accurately identify corresponding parts of figures

Accessing Prior Knowledge

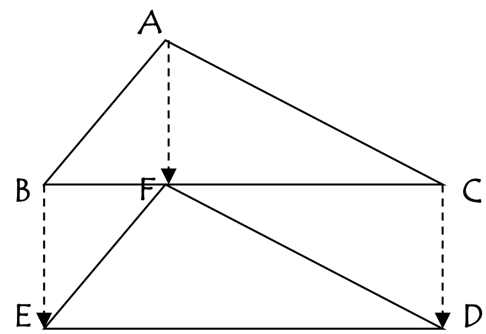
Consider two triangles, △ABC and △FDE, with vertices $A(1,9), B(-3,2),\ C(1,2),\ D(2,3),$ $E(2,-1)$ & $F(9,-1)$.

Draw a diagram and explain why △ABC ≅ △FDE using transformations.



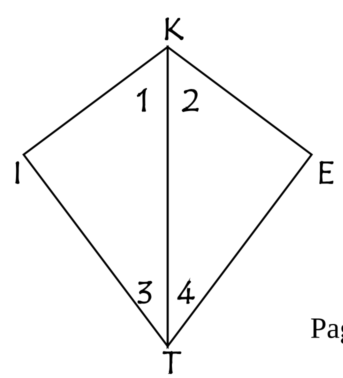
- Congruent Figures
  - Two geometric figures are congruent if one of them could be placed on top of the other and fit exactly, point for point, side for side, and angle for angle.
    - Congruent figures have the same shape & size.

- Congruent Triangles
  - Every triangle has six parts—three angles and three sides.
    - △ABC ≅ △FED
      - <u>Angles</u>
      - <u>Sides</u>



  - <u>Congruent triangles</u>—all pairs of corresponding parts are congruent.

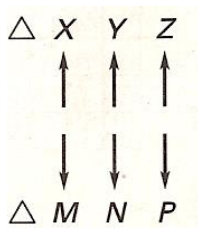
- More About Correspondences
    - △KET is a reflection of △KIT over $\overline{KT}$
      List the parts that reflect onto each other
      - <u>Angles</u>
      - <u>Sides</u>



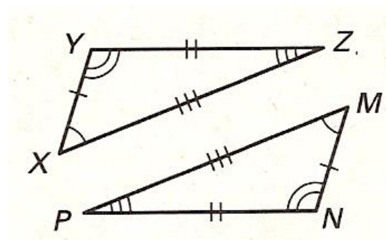
- Whenever a side or an angle is shared by two figures, we can say that the side or angle is congruent to itself.
  - Reflexive Property—Any segment or angle is congruent to itself.

Congruent Parts of Congruent Triangles

- In the diagram below, triangle *XYZ* is congruent to triangle *MNP*.
- Written as: $\triangle XYZ \cong \triangle MNP$. ⟸ CONGRUENCE STATEMENT
  - The notation shows the corresponding vertices and thus the corresponding angles & sides.



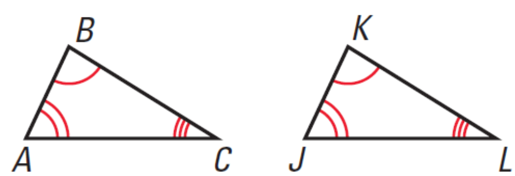
Corresponding Angles



Corresponding Sides

Examples: Determine whether the angles or sides are corresponding angles, corresponding sides, or neither.

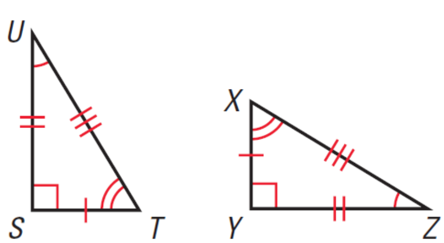
1. $\angle C$ & $\angle L$
2. $\overline{AC}$ & $\overline{JK}$
3. $\overline{BC}$ & $\overline{KL}$
4. $\angle B$ & $\angle L$



Given that $\triangle XYZ \cong \triangle EFD$, determine the congruent side or angle that corresponds to the side or angle.

5. $\angle Y$
6. $\angle D$
7. $\overline{ZX}$
8. $\overline{FD}$

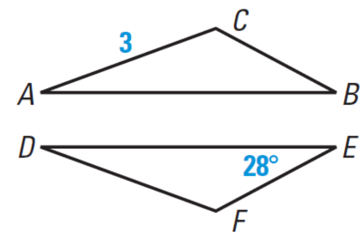
Example 9: Given that $\triangle STU \cong \triangle YXZ$, list all corresponding congruent parts.



Angles

Sides

Example 10: Given $\triangle ABC \cong \triangle DEF$, find the length of $\overline{DF}$ and $m\angle B$.

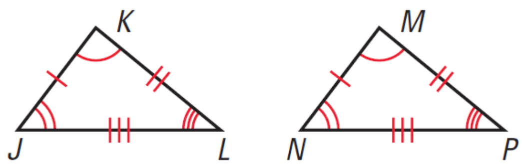


Example 11: Which congruence statement is correct? Why?

A. $\triangle JKL \cong \triangle MNP$

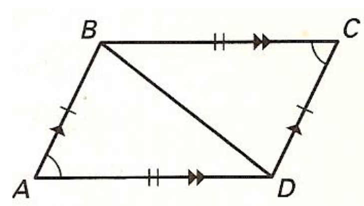
B. $\triangle JKL \cong \triangle NMP$

C. $\triangle JKL \cong \triangle NPM$



Determine Whether Triangles are Congruent

- In the figure, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$
  - Determine whether two triangles are congruent. If they are, write a congruence statement.
    - Remember that when parallel lines are cut by a transversal, the alternate interior angles are congruent.
  - Start by listing the side(s) &/or angles you know to be congruent.
  - Then list any info you can deduce from the figure.

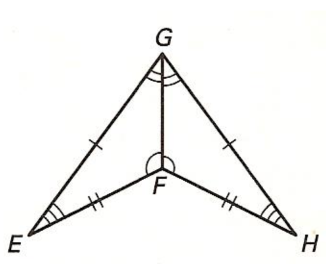


Be on the lookout for vertical angles and shared sides of both triangles.

<u>Examples: Determining Congruence</u>

Determine whether the triangles are congruent. If they are, write a congruence statement.

12.



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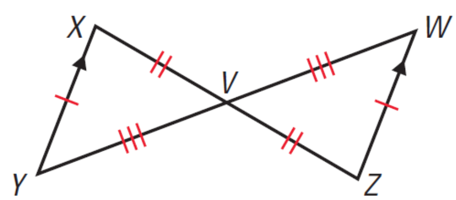


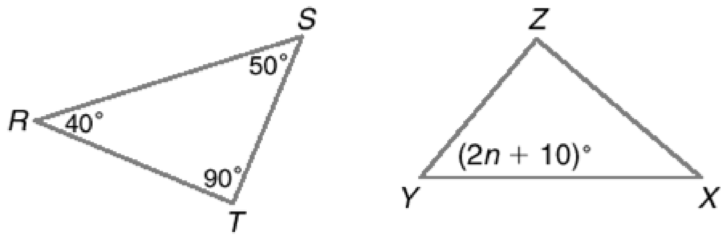
Example 14: Given that $\triangle FGH \cong \triangle TSR$, determine which congruence statement <u>does not</u> describe the triangles.

a. $\triangle HGF \cong \triangle RST$ b. $\triangle FHG \cong \triangle TRS$ c. $\triangle GFH \cong \triangle SRT$

Examples: Geo-Gebra

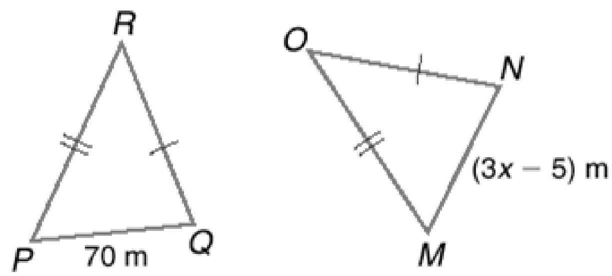
15. If $\triangle RST \cong \triangle XYZ$, find the value of *n*.

To do this problem you must identify the angle congruent to $\angle Y$.



16. If $\triangle RQP \cong \triangle ONM$, find the value of *m*.

To do this problem you must identify the side congruent to $\overline{MN}$.



17. Given: $\triangle ABC \cong \triangle DEF$, AB = 15, BC = 20, AC = 25 & EF = 3x − 7

Find: x

~~~~~~

4.2 Methods of Proving Triangles Congruent

Objectives: Identify included angles & sides

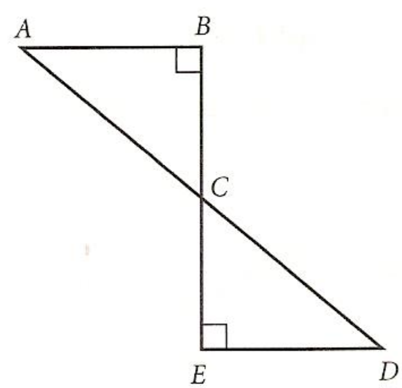
Apply the SSS, SAS, ASA, & AAS postulates

Accessing Prior Knowledge

a. If *C* is the midpoint of $\overline{BE}$, then what two segments are congruent?

b. If $\overline{BE}$ & $\overline{AD}$ intersect at *C*, what two angles must be congruent and why?

c. Name two other congruent angles and explain why they are congruent.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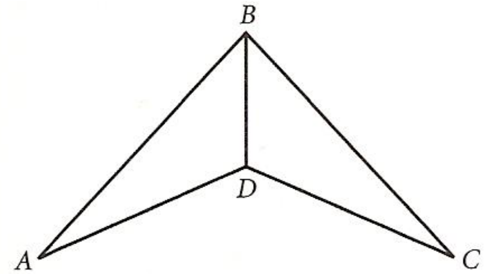
d. If $\overrightarrow{BD}$ bisects $\angle ABC$, then what two angles are congruent?

e. Why is $\overline{BD} \cong \overline{BD}$?



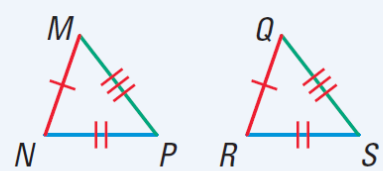
Proving triangles congruent could be a very tedious task if we had to verify the congruence of every one of the six pairs of corresponding parts.

Triangles have some special properties that will enable us to prove two triangles are congruent by comparing only three specially chosen pairs of corresponding parts.

Side-Side-Side Congruence Postulate (SSS)

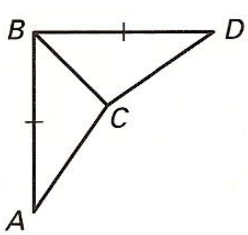
Words If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

Symbols If Side $\overline{MN} \cong \overline{QR}$, and
Side $\overline{NP} \cong \overline{RS}$, and
Side $\overline{PM} \cong \overline{SQ}$,
then $\triangle MNP \cong \triangle QRS$.

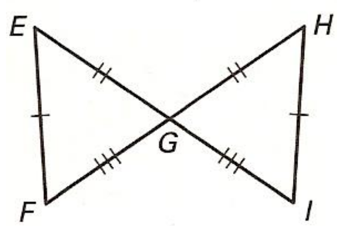


Examples: Using the SSS Congruence Postulate

Does the diagram give enough information to use the SSS Congruence Postulate? Explain your reasoning.

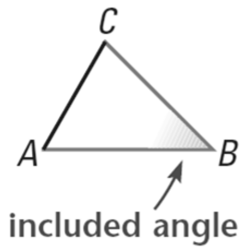
1. 

List those angles &/or sides you know to be congruent:

2. 

List those angles &/or sides you know to be congruent:

- The "Included" Angle
  - In $\triangle ABC$, $\angle B$ is the <u>included angle</u> between sides $\overline{AB}$ & $\overline{BC}$.



Example 3: Identifying the Included Angle

Use the diagram to name the included angle between the two given sides.

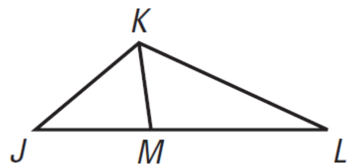
a. $\overline{JK}$ & $\overline{KM}$

b. $\overline{JK}$ & $\overline{MJ}$

c. $\overline{KL}$ & $\overline{JL}$

d. $\overline{KM}$ & $\overline{LM}$

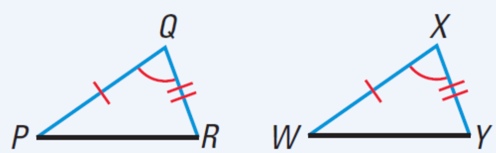
e. $\overline{LK}$ & $\overline{KM}$



Side-Angle-Side Congruence Postulate (SAS)

Words If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

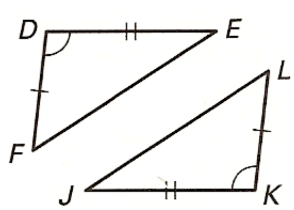
Symbols If Side $\overline{PQ} \cong \overline{WX}$, and
Angle $\angle Q \cong \angle X$, and
Side $\overline{QR} \cong \overline{XY}$,
then $\triangle PQR \cong \triangle WXY$.



Examples: Using the SAS Congruence Postulate

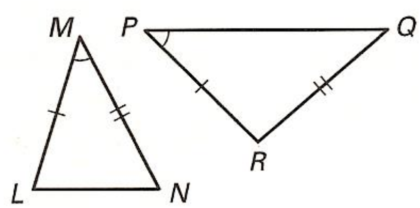
Does the diagram give enough information to use the SAS Congruence Postulate? Explain your reasoning.

4.



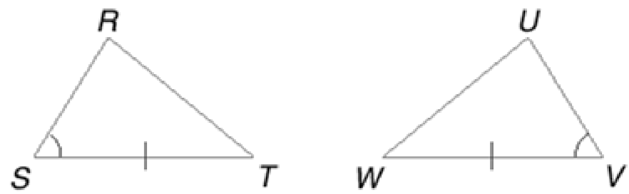
List those angles &/or sides you know to be congruent:

5.



List those angles &/or sides you know to be congruent:

6. What additional information is needed to prove the two triangles are congruent by the SAS Congruence Postulate?



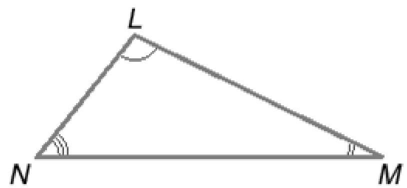
Example 7: Identifying the Included Side

Use the diagram to name the included side between the two given angles.

a. $\angle L$ & $\angle M$

b. $\angle M$ & $\angle N$

c. $\angle L$ & $\angle N$



Angle-Side-Angle Congruence Postulate (ASA)

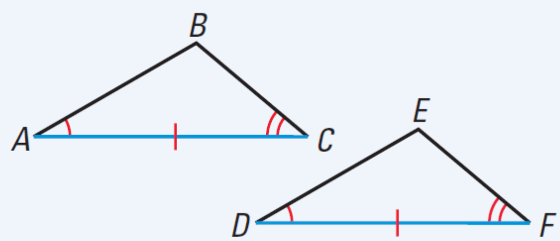
Words If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

Symbols If **A**ngle $\angle A \cong \angle D$, and

Side $\overline{AC} \cong \overline{DF}$, and

Angle $\angle C \cong \angle F$,

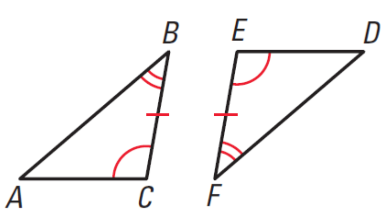
then $\triangle ABC \cong \triangle DEF$.



Examples: Using the ASA Congruence Postulate

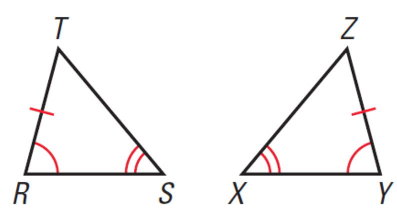
Does the diagram give enough information to use the ASA Congruence Postulate? Explain your reasoning.

8.



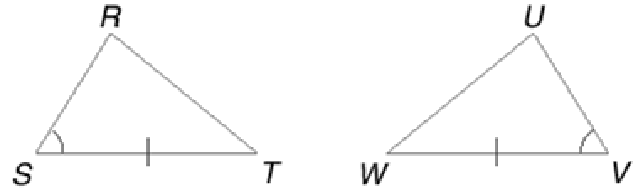
List those angles &/or sides you know to be congruent:

9.

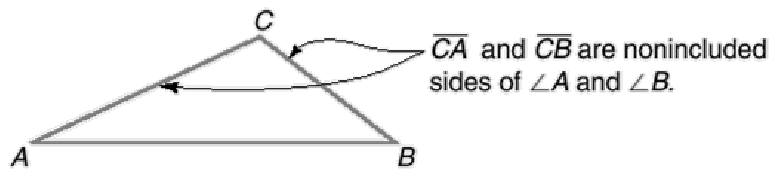


List those angles &/or sides you know to be congruent:

10. What additional information is needed to prove the two triangles are congruent by the ASA Congruence Postulate?



The "Non-Included" Side:



Example 11: Identifying Included & Non-Included Sides

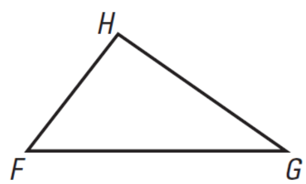
Tell whether the side is *included* or *not included* between the given angles.

$\overline{FG}$ is __?__ between $\angle F$ and $\angle G$.

$\overline{GH}$ is __?__ between $\angle F$ and $\angle G$.

$\overline{FH}$ is __?__ between $\angle H$ and $\angle G$.

$\overline{HG}$ is __?__ between $\angle H$ and $\angle G$.



Angle-Angle-Side Congruence Theorem (AAS)

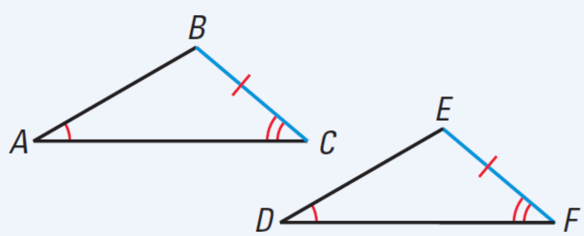
Words If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

Symbols If **A**ngle $\angle A \cong \angle D$, and

Angle $\angle C \cong \angle F$, and

Side $\overline{BC} \cong \overline{EF}$,

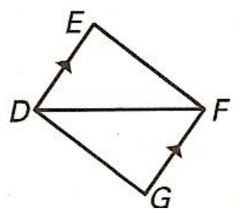
then $\triangle ABC \cong \triangle DEF$.



Examples: Using the AAS Congruence Postulate

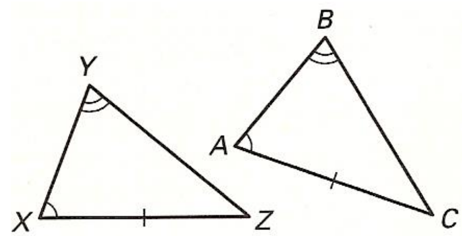
Based on the diagram, can you use the AAS Congruence Theorem to show that the triangles are congruent? If not, what additional congruence is needed?

12.



List those angles &/or sides you know to be congruent:

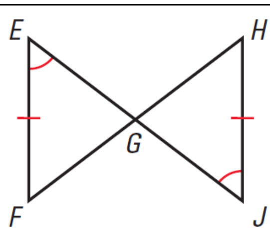
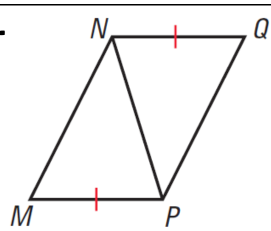
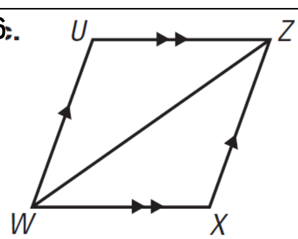
13.



List those angles &/or sides you know to be congruent:

Examples: Deciding Whether Triangles are Congruent

Does the diagram given enough information to show that the triangles are congruent? If so, state the method – SSS, SAS, ASA or AAS – you would use.

Diagram	Congruences	Method
14. 		
15. 		
16. 		

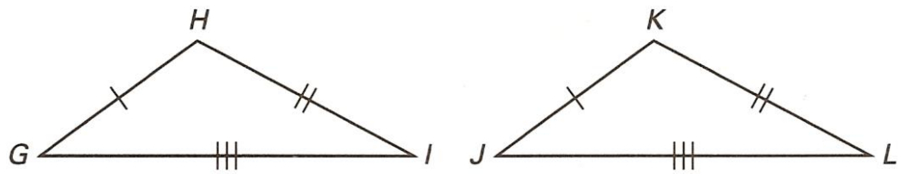
4.3 Proving Triangles Congruent

Objectives: Apply the SSS, SAS, & ASA postulates

Prove that two triangles are congruent in a two-column proof

Structure statements and reasons to formal a logical argument

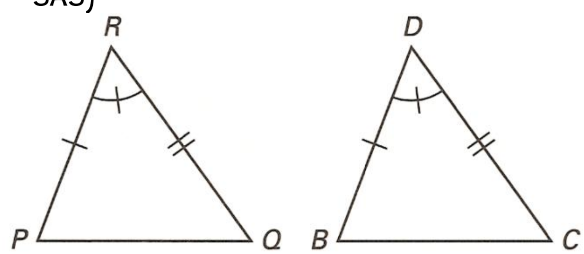
- Methods of Proving Triangles Congruent
  - The SSS Postulate
    - If three sides of one triangle are congruent to the corresponding sides of another triangle, the two triangles are congruent. (Side-Side-Side ~ SSS)



Side | Side | Side

$\overline{GH} \cong$ ______ | ______ $\cong \overline{LJ}$ | $\overline{HI} \cong$ ______

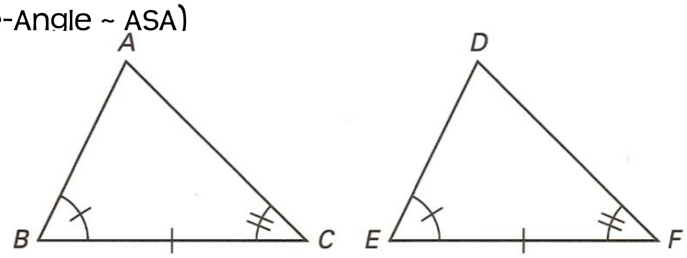
- The SAS Postulate
  - If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, the two triangles are congruent. (Side-Angle-Side ~ SAS)



Side | Angle | Side

$\overline{RP} \cong$ ______ | ∠QRP ≅ ______ | ______ $\cong \overline{DC}$

- The ASA Postulate
  - If two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, the two triangles are congruent. (Angle-Side-Angle ~ ASA)



Angle | Side | Angle

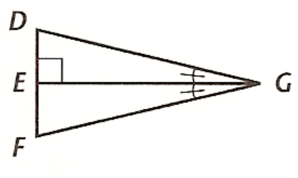
∠CBA ≅ ______ | ______ $\cong \overline{EF}$ | ______ ≅ ∠DFE

Be on the lookout for: (1) vertical angles & (2) shared sides – reflexive property

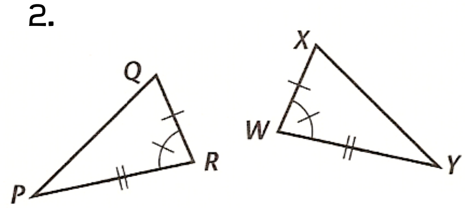
Examples: Congruent? SSS, SAS or ASA?

Determine whether you could prove that the triangles are congruent. If so, write a congruence statement & identify the postulate you could use.

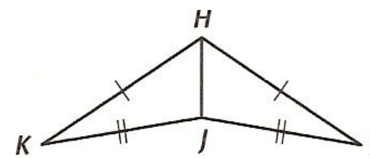
1.



2.



3.

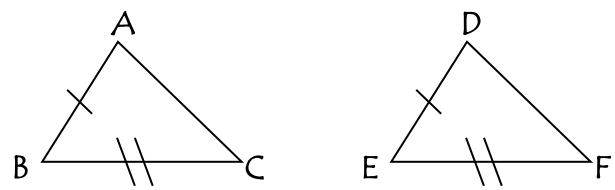


Examples:

In 4 – 7, you are given the congruent angles and sides shown by the tick marks. Name the additional congruent sides or angles needed to prove that the triangles are congruent by each specified method.

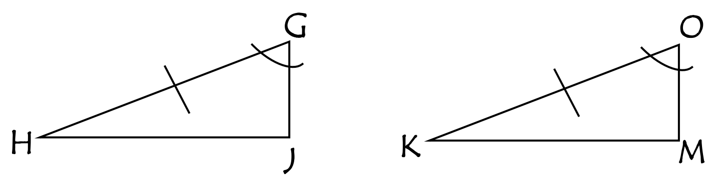
4. a. SSS

b. SAS



5. a. SAS

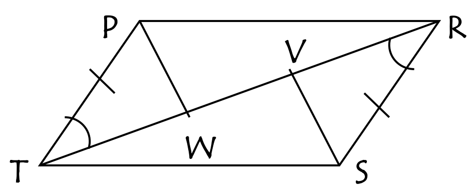
b. ASA



6. Prove: △PWT ≅ △SVR

a. SAS

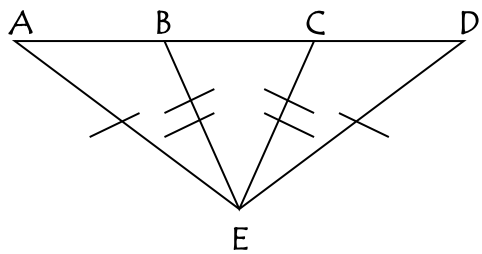
b. ASA



7. Prove: △AEC ≅ △DEB

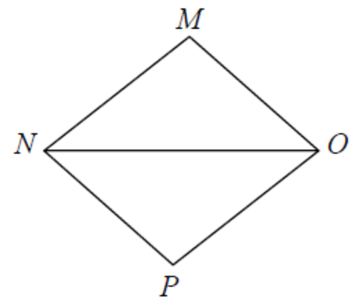
a. SSS

b. SAS



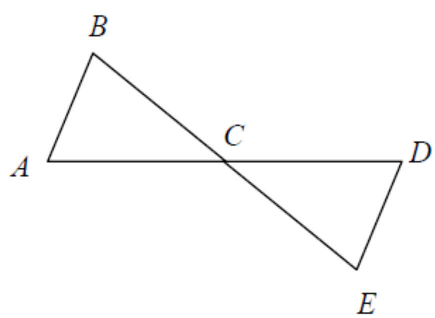
PROOFS:

8. Given: $\overline{MN} \cong \overline{PO}$
$\overline{MO} \cong \overline{PN}$
Prove: $\triangle MNO \cong \triangle PON$



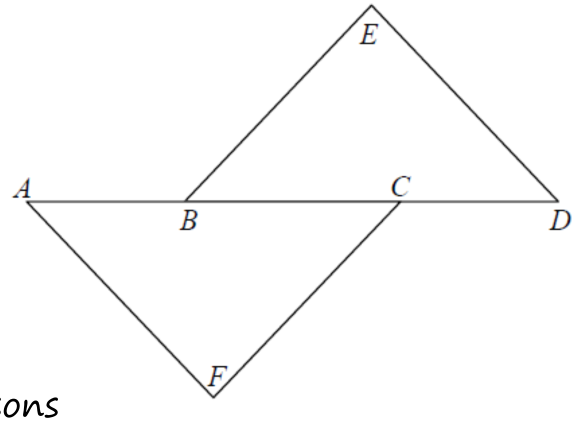
Statements	Reasons

9. Given: $\overline{AC} \cong \overline{DC}$
$\overline{BC} \cong \overline{CE}$
Prove: $\triangle ABC \cong \triangle DEC$



Statements	Reasons

10. Given: $\overline{AB} \cong \overline{CD}$
$\overline{AF} \cong \overline{DE}$
$\angle A \cong \angle D$
Prove: $\triangle FAC \cong \triangle EDB$



Statements	Reasons

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4.4 CPCTC & Circles

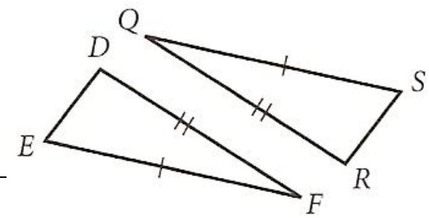
Objectives: Apply the principle of CPCTC and AAS
Apply properties of circles and their radii
Prove that two triangles are congruent in a two-column proof
Structure statements and reasons to formal a logical argument

- Congruent triangles—All pairs of corresponding parts are congruent

- CPCTC
  - "Corresponding Parts of Congruent Triangles are Congruent"
    - Can only be used **AFTER** two triangles have been proven to be congruent.
  - Corresponding parts refers to the matching angles & sides of the respective triangles
~~~~~~

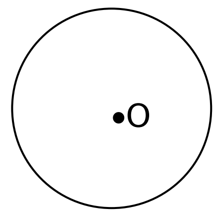
Example: Identifying "CPCT"

1. Given: $\triangle QRS \cong \triangle FDE$ (diagram as shown)
   Identify all pairs of corresponding parts.

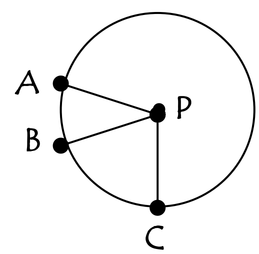


Corresponding Sides	Corresponding Angles

- Circles
  - Point O—Center of the circle shown
    - Every point of the circle is the same distance from the center.
  - A circle is named by its center; this circle is called circle O (or $\odot O$)



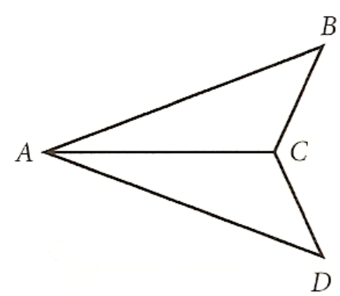
- Radii
  - Points A, B, and C lie on circle P ($\odot P$)
    - $\overline{PA}$ is called a radius
    - $\overline{PA}, \overline{PB}, \& \overline{PC}$ are called radii



Theorem: All radii of a circle are congruent.

PROOFS:

2. Given: $\overline{CB} \cong \overline{CD}$
   $\angle ACB \cong \angle ACD$
   Prove: $\angle B \cong \angle D$

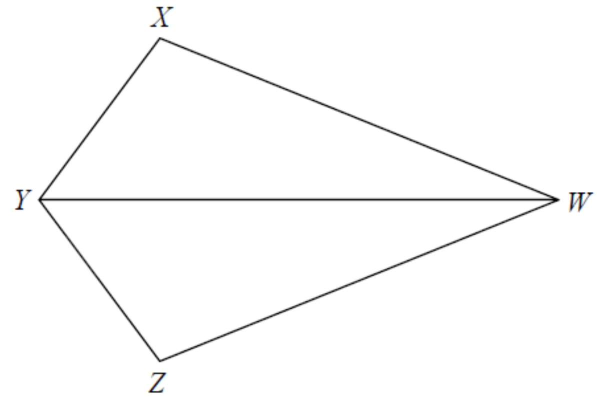


Statements	*Reasons*

3. Given: $\overline{YX} \cong \overline{YZ}$

$\angle XYW \cong \angle ZYW$

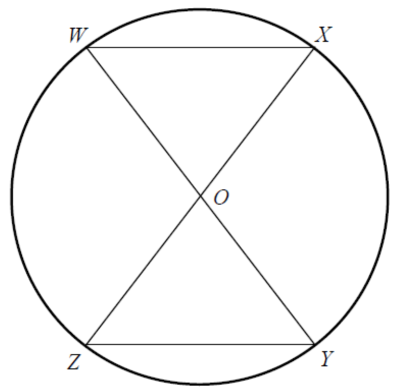
Prove: $\overline{XW} \cong \overline{ZW}$



Statements	Reasons

4. Given: $\odot O$

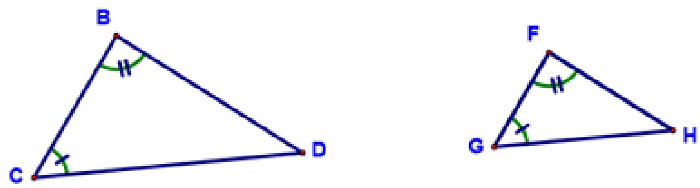
Prove: $\overline{XW} \cong \overline{ZY}$



Statements	Reasons

No-Choice Theorem

- If two angles of one triangle are congruent to two angles of a second triangle, then the third angles are congruent.



Given: $\angle C \cong \angle G$

$\angle B \cong \angle F$

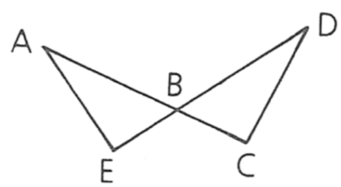
Conclusion: $\angle D \cong \angle H$

- The two triangles need not be congruent for use to apply the No-Choice Theorem.

A PROOF:

5. Given: $\angle A \cong \angle D$

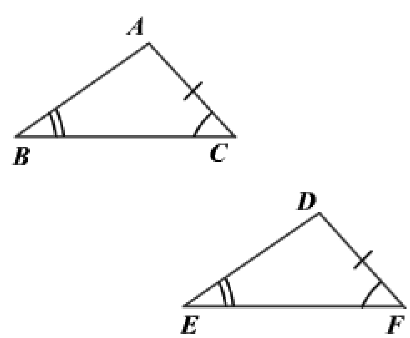
Prove: $\angle E \cong \angle C$



Statements	Reasons

Angle-Angle-Side Theorem (AAS)

- If there exists a correspondence between the vertices of two triangles such that two angles and a non-included side of one are congruent to the corresponding parts of another, then the triangles are congruent.



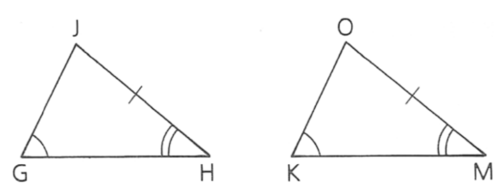
A PROOF:

6. Given: $\angle G \cong \angle K$

$\angle H \cong \angle M$

$\overline{JH} \cong \overline{OM}$

Prove: $\triangle GHJ \cong \triangle KMO$

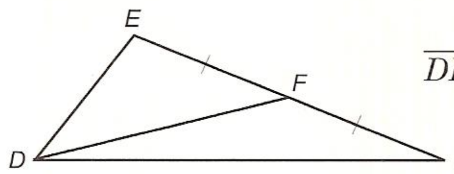


Statements	Reasons

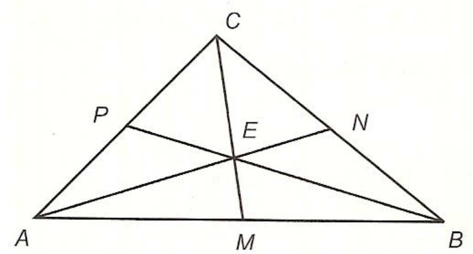
4.5 Beyond CPCTC

Objectives: Identify medians & altitudes of triangles.
Understand why auxiliary lines are used in some proofs.
Write proofs involving steps beyond CPCTC.

- Medians of Triangles
  - A median of a triangle is a line segment drawn from any vertex of the triangle to the midpoint of the opposite side.



$\overline{DF}$ is a median of $\triangle DEC$.



- Every triangle has three medians.
- A median divides (or bisects) the side to which it is drawn into two congruent segments.

Identify the median & the congruent segments formed in the diagram:

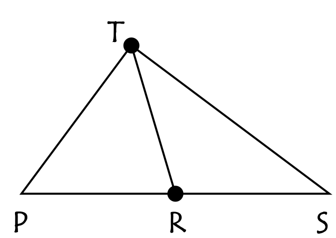
Median	Congruent Segments
$\overline{AN}$	
$\overline{BP}$	
$\overline{CM}$	

- "Definition of Median"
  - Given: median
    - Conclusion: two congruent segments
    - A median bisects the side (of a triangle) to which it is drawn.
  - Prove: median
    - If a segment from a vertex of a △ divides the opposite side into two ≅ segments, it is a median.

.

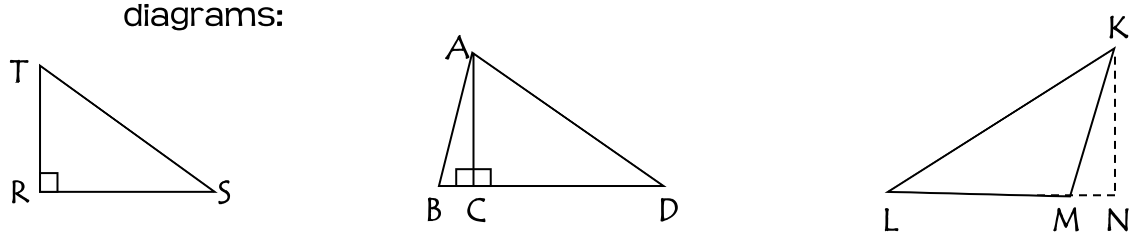
<u>Example: Using Medians</u>

1. Given: $\overline{TR}$ is the median to $\overline{PS}$, PR = x + 8, PT = 2x – 1, RS = 2x – 6
   Find: PT



Altitudes of Triangles

- An altitude of a triangle is a line segment drawn from any vertex of the triangle to the opposite side, extended if necessary, and perpendicular to that side.
  - Every triangle has three altitudes.
  - An altitude of a triangle forms right angles with one of the sides.
    - Identify the altitude shown & the right angles formed in the following diagrams:



△TRS – altitude: ______	△ABD – altitude: ______	△KLM – altitude: ______
right angle(s): ________	right angle(s): ________	right angle(s): ________

"Definition of Altitude"

- Given: altitude
  - Conclusion: right angle(s)
  - An altitude of a △ form right ∠s with the side to which it is drawn.
- Prove: altitude
  - If a segment from a vertex of a △ forms right ∠s with the opposite side, it is an altitude.

Auxiliary Lines

- Need there to be line connecting two points? No problem!
  - Auxiliary lines connect two points already in the diagram.

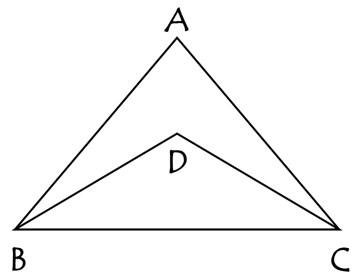
Whenever we use an auxiliary line in a proof, we must be able to show that such a line can be drawn & then justify it with the following postulate:

Two points determine a line.

Statements	*Reasons*
⋮	⋮
Draw $\overline{AL}$	Two points determine a line.

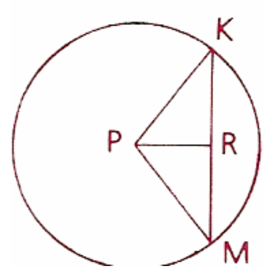
PROOFS:

2. Given: $\overline{AB} \cong \overline{AC}$
$\overline{BD} \cong \overline{CD}$
Prove: $\triangle ABD \cong \triangle ACD$



Statements	Reasons

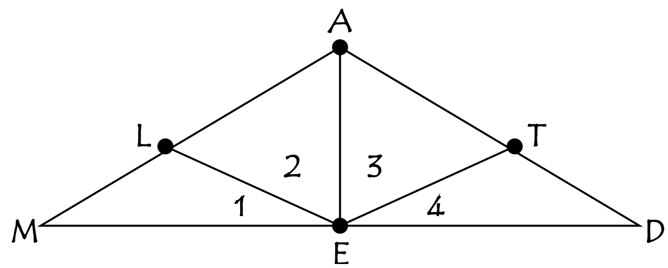
3. Given: ⊙P
$\overrightarrow{PR}$ bisects $\angle KPM$
Prove: $\overline{PR}$ is a median



Statements	Reasons

Could the altitude of a triangle be a median as well?

4. Given: $\overline{AE}$ is an altitude of $\triangle MAD$
$\angle 2 \cong \angle 3$
$\overline{EL} \cong \overline{ET}$
$\angle MLE \cong \angle DTE$
Prove: $\overline{AE}$ is the median to $\overline{MD}$

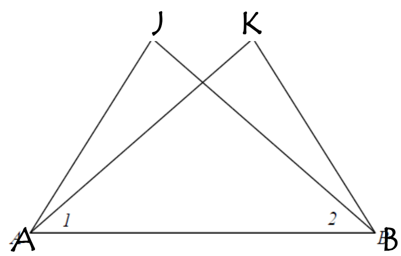


Statements	Reasons

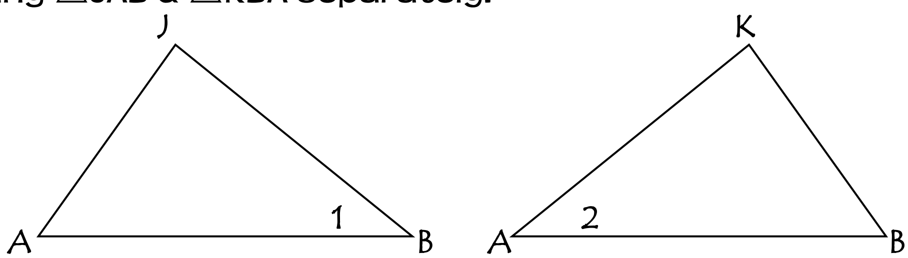
4.6 Overlapping Triangles

Objectives: Use overlapping triangle in proofs
Prove that two triangles are congruent in a two-column proof
Structure statements and reasons to formal a logical argument

Given: $\angle JAB \cong \angle KBA$
$\angle 1 \cong \angle 2$
Prove: $\angle J \cong \angle K$



- What triangles would I prove to be congruent?
- What postulate would I use here?
- Try drawing △JAB & △KBA separately.



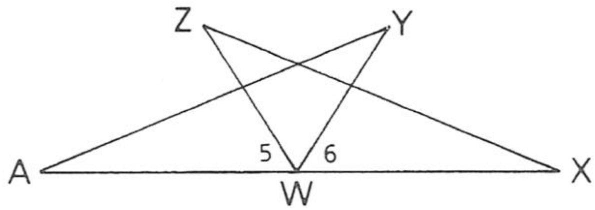
Given: $\angle JAB \cong \angle KBA$
$\angle 1 \cong \angle 2$
Prove: $\angle J \cong \angle K$

Statements	*Reasons*

Helpful Hints w/Overlapping Triangles

- Draw the triangles separately.
- Outline the two triangles in different colors.
- ALSO…there will be a reflexive step—that shared side or angle.

1. Given: $\overline{YW}$ bisects $\overline{AX}$
$\angle A \cong \angle X$
$\angle 5 \cong \angle 6$
Prove: $\overline{ZW} \cong \overline{YW}$



Statements	*Reasons*

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4.7 Triangles in Proofs

Objectives: Use isosceles, equilateral, and right triangles in proofs
Prove that two triangles are congruent in a two-column proof
Structure statements and reasons to formal a logical argument

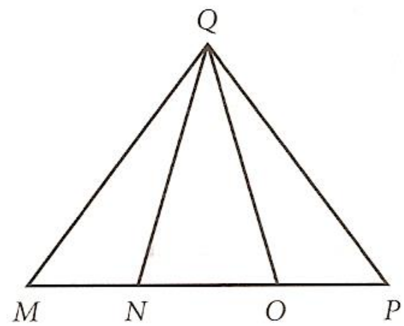
- Triangles in Proofs
  - Isosceles Triangles
    - If at least two sides of a triangle are congruent, then the triangle is isosceles.
  - Equilateral Triangles
    - If all sides of a triangle are congruent, then the triangle is equilateral.
  - Right Triangles
    - If a triangle has a right angle, then it is a right triangle.
~~~~~~

Isosceles Triangle Theorems

- If △, then △.
  - If two sides of a triangle are congruent, then the angles opposite the sides are congruent.
- If △, then △.
  - If two angles of a triangle are congruent, then the sides opposite the angles are congruent.

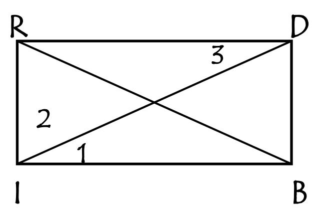
PROOFS:

1. Given: $\overline{QM} \cong \overline{QP}$
   $\overline{MN} \cong \overline{PO}$
   Prove: $\angle QNP \cong \angle QOM$



Statements	Reasons

2. Given: $\overline{BI} \cong \overline{RD}$
   $\overline{RI} \cong \overline{BD}$
   $\angle 3$ is comp. to $\angle 2$
   Prove: $\triangle RIB$ is a right $\triangle$

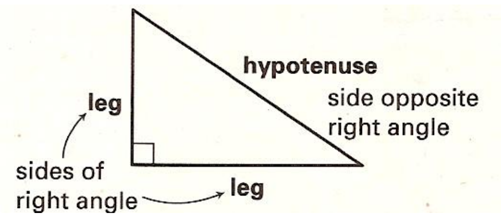


Statements	Reasons

4.8 HL Postulate

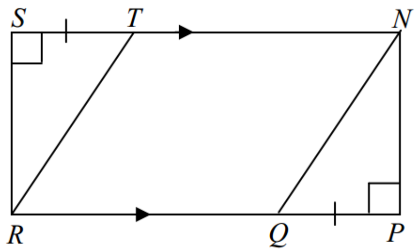
Objectives: Use the HL postulate to prove right triangles are congruent
Prove that two triangles are congruent in a two-column proof
Structure statements and reasons to formal a logical argument

- The Hypotenuse-Leg Postulate
  - If the hypotenuse & a leg of one right triangle are congruent to the corresponding parts of another right triangle, the triangles are congruent. (HL)

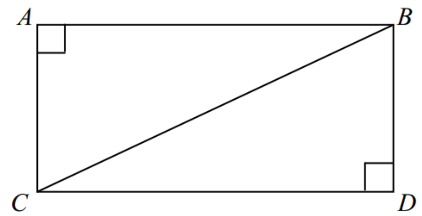


Examples: What additional information would you need to prove the triangles congruent by the HL theorem?

1. $\triangle STR \cong \triangle PQN$

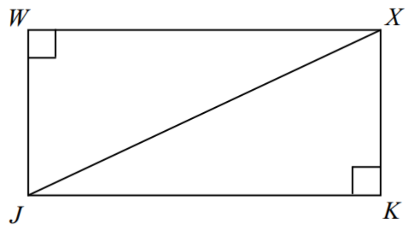


2. $\triangle ABC \cong \triangle DCB$



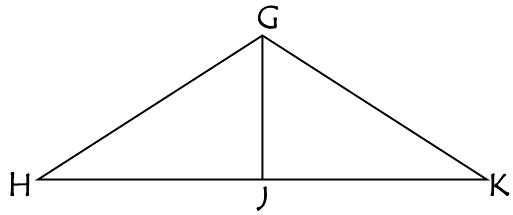
PROOFS:

3. Given: $\overline{WJ} \cong \overline{KX}$
$\angle JWX$ is a right angle
$\angle XKJ$ is a right angle
Prove: $\triangle WJX \cong \triangle KJX$



Statements	Reasons

4. Given: $\overline{GH} \cong \overline{GK}$
$\overline{GJ}$ is an altitude
Prove: $\overrightarrow{GJ}$ bisects $\angle HGK$



Statements	*Reasons*

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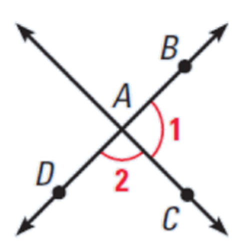
4.9 Perpendicular Bisectors & Equidistance

Objectives: Recognize the relationship between equidistance and perpendicular bisectors
Prove that two triangles are congruent in a two-column proof
Structure statements and reasons to formal a logical argument

A Right-Angle Theorem

- If two angles are both supplementary and congruent, then they are right angles.

Given: $\angle 1 \cong \angle 2$
$\angle 1$ & $\angle 2$ form a linear pair
Conclusion: $\angle 1$ & $\angle 2$ are right angles



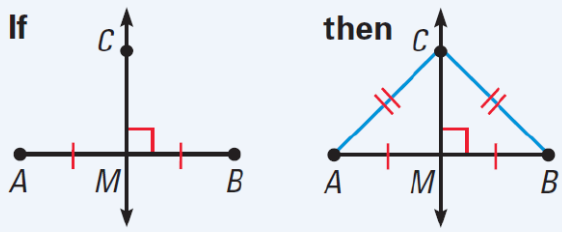
Perpendicular Bisector

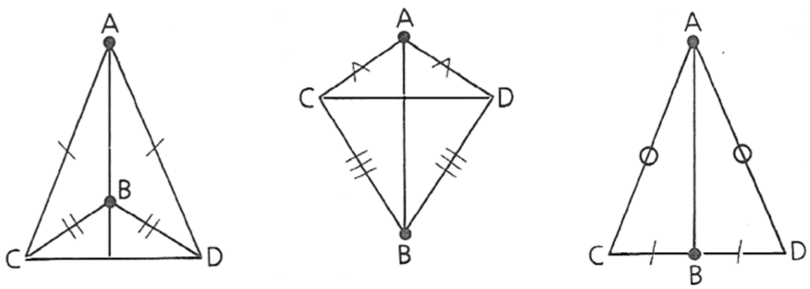
- The perpendicular bisector of a segment is the line that bisects and is perpendicular to the segment.
~~~~~~

Perpendicular Bisector Theorem

Words If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Symbols If C is on the perpendicular bisector of $\overline{AB}$, then $\overline{CA} \cong \overline{CB}$.





Point A & point B are equidistance from the endpoints C & D of $\overline{CD}$.

$\overleftrightarrow{AB}$ is the perpendicular bisector of $\overline{CD}$.

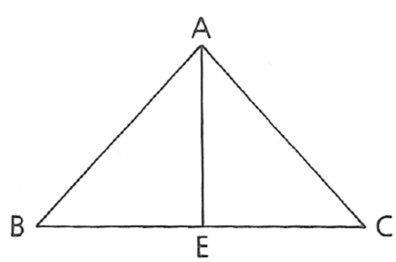
- Equidistance Theorems:
  - If two points are each equidistant from the endpoints of a segment, then the two points determine the perpendicular bisector of that segment.

PROOFS:

Prove that…The line joining the vertex of an isosceles triangle to the midpoint of the base is perpendicular to the base.

1. Given: $\triangle ABC$ is isosceles, with $\overline{AB} \cong \overline{AC}$; E is the midpoint of $\overline{BC}$

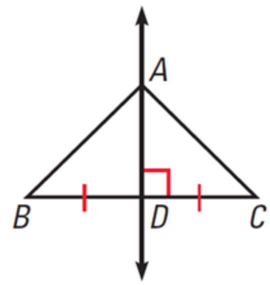
   Prove: $\overleftrightarrow{AE} \perp \overline{BC}$



Statements	Reasons

2. Given: $\overleftrightarrow{AD}$ is the $\perp$ bisector of $\overline{BC}$

Prove: $\overline{AB} \cong \overline{AC}$

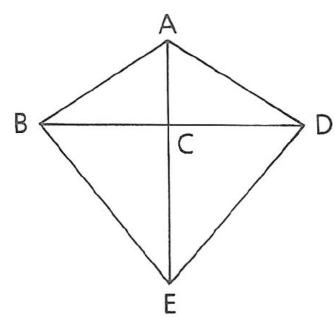


Statements	Reasons

3. Given: $\overline{AB} \cong \overline{AD}$

$\overline{BC} \cong \overline{CD}$

Prove: $\overline{BE} \cong \overline{ED}$



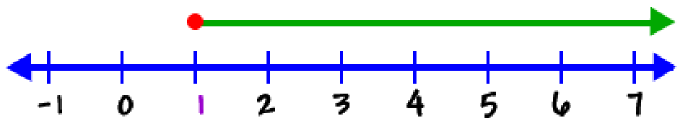
Statements	Reasons

Algebra Review ~ Inequalities

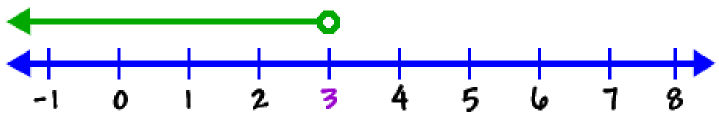
Objective: Solve an inequality using the addition and multiplication principles and then graph the solution set

- Inequalities
  - An inequality is any sentence containing $<, >, \leq$, or $\geq$.
    - Solution ~ any replacement for the variables that makes an inequality true
    - Solution Set ~ the set of all solutions
  - Graphing Inequalities:

$x \geq 1$



$x < 3$



Schultz says:

SOLVE INEQUALITIES AS YOU WOULD EQUATIONS. . .

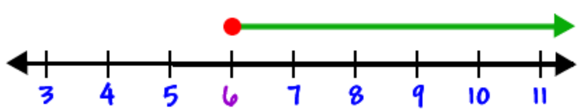
. . .BUT BE CAREFUL WHEN MULTIPLYING OR DIVIDING BY A NEGATIVE NUMBER.

$$2x - 5 \geq 7$$
$$+5 \quad +5$$ **ditch the 5**
$$2x \geq 12$$
$$\frac{2x}{2} \geq \frac{12}{2}$$ **ditch the 2**
$$x \geq 6$$

OK, so what does this answer mean?

(It's super important in math to understand what your answers mean!)

We can graph it on a number line:



So, in our original problem, $2x - 5 \geq 7$ **, X can be 6... or X can be a number bigger than 6.**

Schultz says:

SOLVE INEQUALITIES AS YOU WOULD EQUATIONS…

…BUT BE CAREFUL WHEN MULTIPLYING OR DIVIDING BY A NEGATIVE NUMBER.

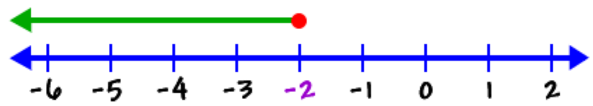
Solve

$$-3x \le 6$$

$$\frac{-3x}{-3} \le \frac{6}{-3} \quad \text{ditch the } -3$$

$$x \le -2$$

It looks ok ... But, is it?



This means that X can be -2 or any other number less than -2.

Let's check!

$$-3x \le 6$$

$$x = -2 \longrightarrow -3(-2) \le 6$$

$$6 \le 6 \quad \text{Yep - that works.}$$

$$x = -4 \longrightarrow -3(-4) \le 6$$

$$12 \le 6 \quad \text{NO WAY, DUDE!}$$

It didn't work. Wazzup with that?

Here's the freaky thing:

When you divide (or multiply) by a negative number, you mess up the inequality sign!

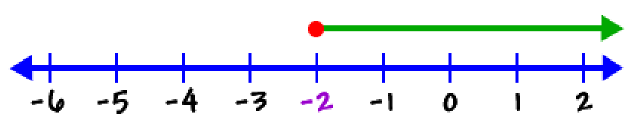
When you multiply or divide an inequality by a negative number, FLIP THE SIGN!

$$-3x \le 6$$

$$\frac{-3x}{-3} \ge \frac{6}{-3}$$

alert! ↓ divide by -3 and flip the sign

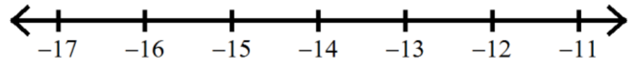
$$x \ge -2$$



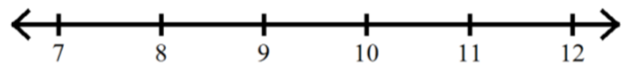
EXAMPLES

Solve each inequality and graph the solution set.

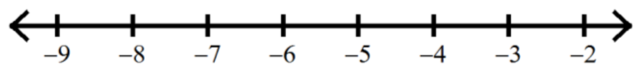
1. $8n - 4 < -116$



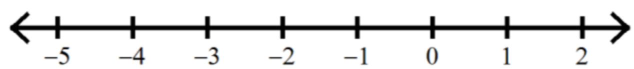
2. $6 - 6x \leq -54$



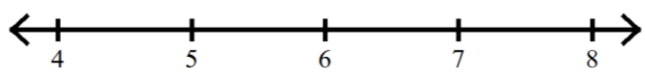
3. $\frac{m}{3} + 4 > 2$



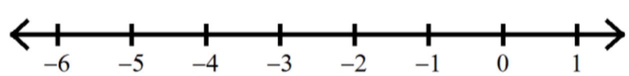
4. $\frac{y - 5}{2} > -4$



5. $-x - 4 \geq 14 - 4x$



6. $4(x + 2) > 1 - 3x$

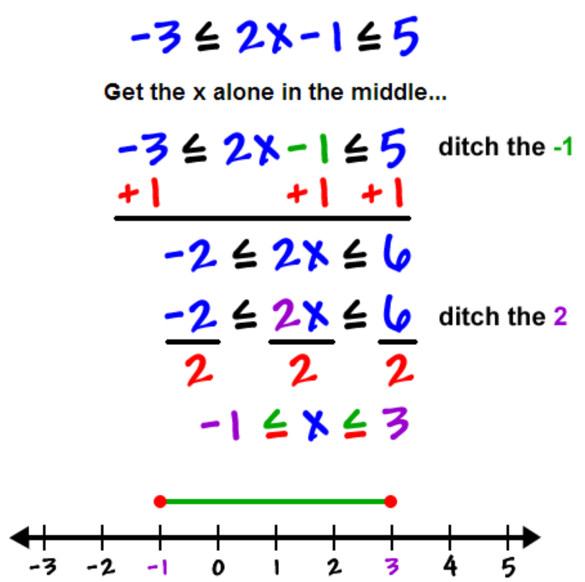


Algebra Review ~ Compound Inequalities

Objective: Solve compound inequalities and graph the solution set

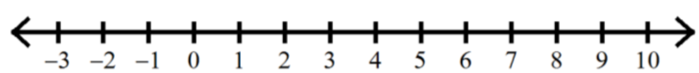
- Compound Inequality
  - Consist of two or more inequalities joined by the word "and" or the word "or"
  - Conjunction
    - When two or more sentences are joined by the word "and"
    - $-2 < x \text{ and } x < 1 \leftrightarrow -2 < x < 1$
    - The solution set of a conjunction is the intersection of the solution sets.

Example:



Your turn:

$$3 < 3 + 4x \leq 31$$

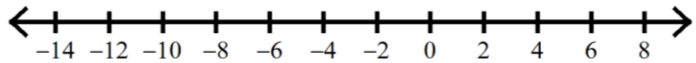


- Compound Inequality
  - Consist of two or more inequalities joined by the word "and" or the word "or"
  - Disjunction
    - When two or more sentences are joined by the word "or"
    - $-2 < x \text{ or } x < 1$
    - The solution set of a disjunction is the union of the individual solution sets.

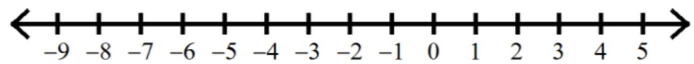
EXAMPLES

Solve each inequality and graph the solution set.

2. $-7x \leq -28$ or $x + 3 < -7$



3. $\frac{n}{3} \leq -2$ or $n - 5 > -4$



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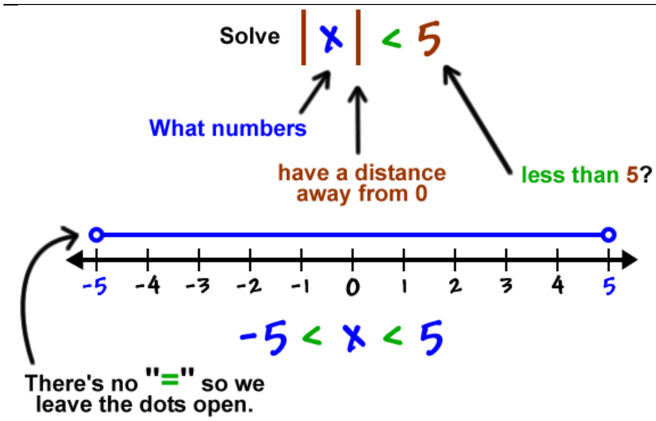
Algebra Review ~ Absolute Value Inequalities

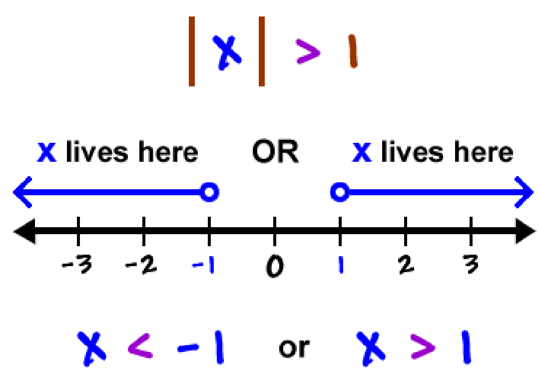
Objective: Solve inequalities with absolute value expressions and graph the solution set

- Absolute Value Inequalities
  - For all real numbers a & b, b > 0, the following statements are true:
    - If $|a| < b$, then $-b < a < b$
    - If $|a| > b$, then $a > b$ or $-a > b$

$|a| < b$



$|a| > b$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How to Solve Absolute Value Inequalities

1. Isolate the absolute value expression on the left side of the inequality.
2. Rewrite as two inequalities—sans absolute value bars (Don't change the inequality symbols.)
    a. a.v. expression < value
    b. –(a.v. expression) < value
        i. You're negating the a.v. expression in the second inequality.
3. Solve each inequality.
4. Graph the solution set.

Some absolute value inequalities have *no solutions.*

- i.e. |4x – 9| < -7 is *never* true.
    - Since the absolute value of a number is always positive or zero, there is *no* replacement for x that will make the sentence true.
        - So *less than* a negative number is *never* true.

Some absolute value inequalities are *always* true.

- i.e. |10x + 3| > -5 is *always* true.
    - Since the absolute value of a number is always positive or zero, *any* replacement for x will make the sentence true.
        - So *greater than* a negative number is *always* true.

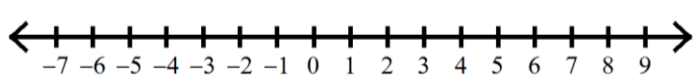
EXAMPLES

Solve each inequality and graph the solution set.

STEP 1: ISOLATE STEP 2: REWRITE STEP 3: SOLVE

1. $|9n - 6| - 6 \geq 24$

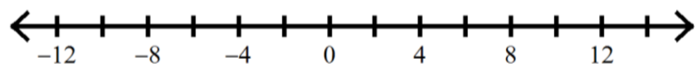
STEP 4: GRAPH



STEP 1: ISOLATE STEP 2: REWRITE STEP 3: SOLVE

2. $-2|3-2a| \leq -34$

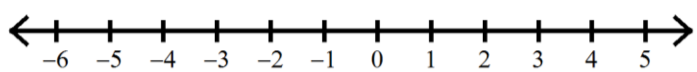
STEP 4: GRAPH



STEP 1: ISOLATE STEP 2: REWRITE STEP 3: SOLVE

3. $\dfrac{|6-10m|}{10} \geq 2$

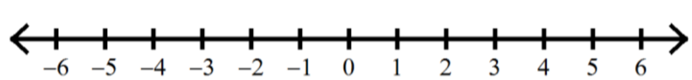
STEP 4: GRAPH



STEP 1: ISOLATE STEP 2: REWRITE STEP 3: SOLVE

4. $3|6x-6|+2 \leq 74$

STEP 4: GRAPH



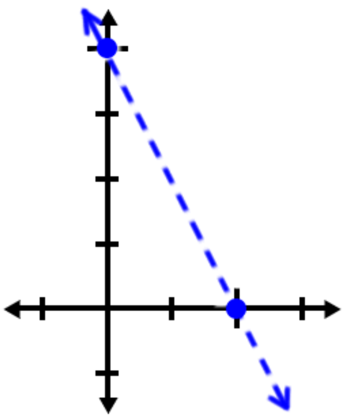
Algebra Review ~ Linear Inequalities

Objective: Graph linear inequalities in two variables

Graph $2x + y > 4$

Solution:

First, do the line:



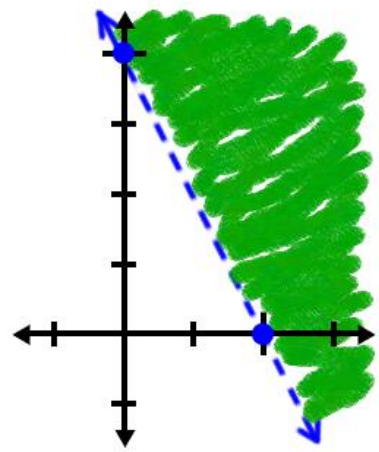
We make it a dashed line since there is no =.

So, which side do we shade?

Let's try (0 , 0):

$$2x + y > 4$$
$$2(0) + 0 > 4$$
$$0 > 4$$

Nope!



All the points in the shaded region work in

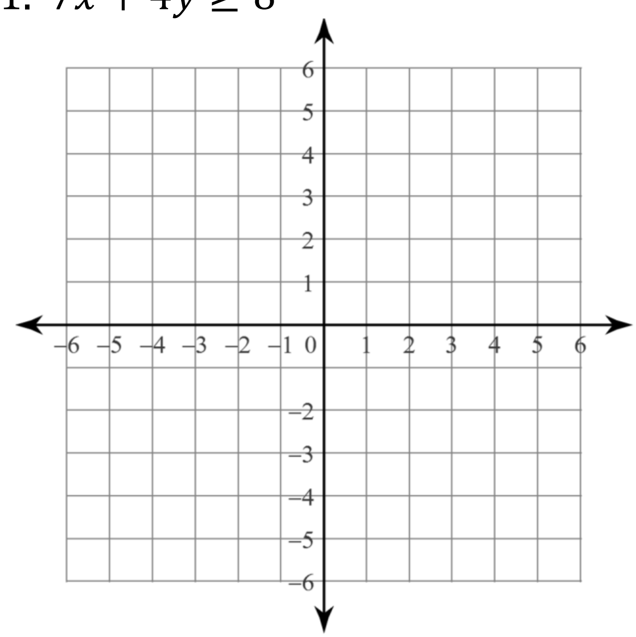
$$2x + y > 4$$

(0 , 0) doesn't work! That means nothing on that side will work ... So, all the good points must be on the other side!

EXAMPLES

Graph each linear inequality.

1. $7x + 4y \geq 8$



2. $x + 5y < 10$