3.2 TRIANGLES & POINTS OF CONCURRENCY

△ Triangle Types

➢ A triangle is a polygon with three sides.

➢ Triangles can be classified in two ways: by their angle measures or by their side lengths.

### NAMES OF TRIANGLES

#### Classification by Sides

- **Equilateral Triangle**: 3 congruent sides
- **Isosceles Triangle**: At least 2 congruent sides
- **Scalene Triangle**: No congruent sides

#### Classification by Angles

- **Acute Triangle**: 3 acute angles
- **Equiangular Triangle**: 3 congruent angles
- **Right Triangle**: 1 right angle
- **Obtuse Triangle**: 1 obtuse angle

*Note: An equiangular triangle is also acute.*

△ Theorems Involving Triangles

➢ **Triangle Angle-Sum Theorem**

- The sum of the measures of the angles of a triangle is 180 degrees.
Unit 3: Triangles

\[ \angle 1 + \angle 2 + \angle 3 = \_\_\_\_\_ \]
\[ \angle 3 + \angle 4 = \_\_\_\_\_ \]
\[ \angle 1 + \angle 2 = \_\_\_\_\_ \]

- Corollary: The acute angles of a right triangle are complementary.
  \[ \angle D + \angle E = 90 \]

➤ Triangle Exterior-Angle Theorem
  - The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

Examples ~

1. In \( \triangle LMN \), \( \angle L = 8x \), \( \angle M = \angle N = 6x - 1 \). Use the Triangle Angle-Sum Theorem to set up and solve an equation to find the value of \( x \).

2. Use the Triangle Exterior-Angle Theorem to set up and solve an equation to find the value of \( x \). Then find \( \angle S \).

♀ Theorems Involving Isosceles Triangles
  - If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
- Converse of Isosceles Triangle Theorem
  If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Examples ~

3. Find \( m\angle F \) and then set up and solve an equation to find the value of \( x \).

4. Set up and solve an equation to find the value of \( y \) and then find \( RS \).

5. \( \triangle ACD \) is isosceles with \( \angle D \) as the vertex angle. \( B \) is the midpoint of \( \overline{AC} \).
   a. Draw the triangle.
   b. \( AB = x + 5, BC = 2x - 3, \) and \( CD = 2x + 6; \) find the perimeter of \( \triangle ACD \).

The Midsegment of a Triangle

- A midsegment of a triangle is a segment that joins the midpoints of two sides of the triangle.
- **Triangle Midsegment Theorem** ~ A midsegment of a triangle is parallel to the third side of the triangle, and its length is half the length of that side.
  - \( DE \parallel AC \)
  - Midsegment = \( \frac{1}{2} \) (third side)
  - Third side = \( 2 \times \) (the midsegment)
Examples ~

Find each measure given that $PM$ & $PN$ are midsegments.

6. $JL$  
7. $PM$  
8. $m\angle MLK$

Set up and solve an equation to find the value of $n$.

9. 

10. 

The Medians & Centroid of a Triangle

- **Median of a Triangle** ~ A segment whose endpoints are a vertex of a triangle and the midpoint of the opposite side

- The medians of a triangle are concurrent at a point – the centroid of the triangle – that is two-thirds the distance from each vertex to the midpoint of the opposite side.
  - Look at median $PK$
  - It has been split into two segments:
    - $PX$: the shorter segment
    - $XK$: the longer segment
    - Median $= 3 \times$ (the shorter segment)
    - The longer segment $= 2 \times$ (the shorter segment)
    - The shorter segment $= \text{median} \div 3$
Examples ~

W is the centroid of \( \triangle RTV \), \( VX = 204 \), and \( RW = 104 \). Find each length.

11. \( VW \)
12. \( WX \)
13. \( RY \)
14. \( WY \)

The Altitudes & Orthocenter of a Triangle

- **Altitude of a Triangle** – The perpendicular segment from a vertex to the line containing the opposite side.
- The lines that contain altitudes of a triangle are concurrent. The point of concurrency is called the **orthocenter** of the triangle.

The Incenter of a Triangle

- The angle bisectors of the vertices of a triangle are concurrent. The point of concurrency is called the incenter of the triangle.

**Theorem 5-2-2** **Incenter Theorem**

The incenter of a triangle is equidistant from the sides of the triangle.

\[ PX = PY = PZ \]
The Circumcenter of the Triangle

- The perpendicular bisectors of the sides of a triangle are concurrent. The point of concurrency is called the circumcenter of the triangle.

**Theorem 5-2-1 Circumcenter Theorem**

The circumcenter of a triangle is equidistant from the vertices of the triangle.

\[ PA = PB = PC \]

The circumcenter can be inside the triangle, outside the triangle, or on the triangle.

Examples ~

\( \overline{DY}, \overline{EY}, \& \overline{FY} \) are the perpendicular bisectors of \( \triangle ABC \).

15. Is \( Y \) the incenter or the circumcenter?

Find each length.

16. \( CF \) \hspace{1cm} 17. \( YC \)

18. \( DB \) \hspace{1cm} 19. \( AY \)

\( \overline{TJ} \) & \( \overline{SJ} \) are angle bisectors of \( \triangle RST \).

20. Is \( J \) the incenter or circumcenter?

Find each measure.

21. The distance from \( J \) to \( \overline{RS} \)

22. \( m\angle RTJ \)
3.3 BEGINNING PROOFS

OBJECTIVES:
- PROVE A CONJECTURE THROUGH THE USE OF A TWO-COLUMN PROOF
- STRUCTURE STATEMENTS AND REASONS TO FORM A LOGICAL ARGUMENT
- INTERPRET GEOMETRIC DIAGRAMS

Why Study Proofs?

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You will need them every day, I hope, without knowing it. Geometry is beautifully logical, and it teaches you how to think and prove that things are so, step by step by step. Proofs are excellent lessons in reasoning. Without logic and reasoning, you are dependent on jumping to conclusions or - worse - having empty opinions.

Assumptions from Diagrams

- You should assume:
  - Straight lines & angles
  - Collinearity of points
  - Betweenness of points
  - Relative positions of points

- You should NEVER assume:
  - Right angles
  - Congruent segments
  - Congruent angles
  - Relatives sizes of segments & angles

Examples ~

1. Should we assume that S, T, and V are collinear in the diagram?

2. Should we assume that $m\angle S = 90$?

3. What can we assume from this diagram?

4. Use that assumption to set up and solve an equation to find $x$.

5. Find $m\angle MTA$
Often, we use identical tick marks to indicate congruent segments and arc marks to indicate congruent angles.

Examples ~

6. Identify the congruent segments and/or angles in each diagram.

   a) ![Diagram A]
   
   b) ![Diagram B]

   c) What kind of triangle is \( \triangle ABC \)? How do you know?

   d) Is \( b \parallel c \)? Explain why or why not.

7. In the diagram below, \( \angle DEG = 80^\circ \), \( \angle DEF = 50^\circ \), \( \angle HJM = 120^\circ \), and \( \angle HJK = 90^\circ \). Draw a conclusion about \( \angle FEG \) & \( \angle KJM \).

   ![Diagram C]
   ![Diagram D]

Writing Two-Column Proofs

- **Proof** – A convincing argument that shows why a statement is true
  - The proof begins with the given information and ends with the statement you are trying to prove.
  - Two-Column Proof:
    
    | Statements | Reasons |
    |------------|---------|
    | • Specific – applies only to this proof | • General – can apply to any proof |
Procedure for Drawing Conclusions

1. Memorize theorems, definitions, and postulates.
2. Look for key words and symbols in the given information.
3. Think of all the theorems, definitions, and postulates that involve those keys.
4. Decide which theorem, definition, or postulate allows you to draw a conclusion.
5. Draw a conclusion, and give a reason to justify the conclusion. Be certain that you have not used the reverse of the correct reason.

- The “If…” part of the reason matches the given information, and the “then…” part matches the conclusion being justified.

Schultz says: We write our reasons—if they are not theorems, postulates, or properties—as “if…then” statements.

Try this thought process:

If what I just said, then what I’m trying to prove.

Theorem — A mathematical statement that can be proved

Theorem: If two angles are right angles, then they are congruent.

Given: \( \angle A \) is a right angle
\( \angle B \) is a right angle

Prove: \( \angle A \cong \angle B \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>1. ( \angle A ) is a right angle</td>
<td>1.</td>
</tr>
<tr>
<td>2. ( m\angle A = 90 )</td>
<td>2.</td>
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<tr>
<td>3. ( \angle B ) is a right angle</td>
<td>3.</td>
</tr>
<tr>
<td>4. ( m\angle B = 90 )</td>
<td>4.</td>
</tr>
<tr>
<td>5. ( \angle A \cong \angle B )</td>
<td>5.</td>
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</table>
Theorem: If two angles are straight angles, then they are congruent.

Given: Diagram as shown.

Prove: \( \angle ABC \cong \angle DEF \)

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<tbody>
<tr>
<td>1. Diagram</td>
<td>1.</td>
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<tr>
<td>2.</td>
<td>2. Assumed from diagram.</td>
</tr>
<tr>
<td>3. ( m\angle ABC = 180 )</td>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
<td>4. Assumed from diagram.</td>
</tr>
<tr>
<td>5. ( m\angle DEF = 180 )</td>
<td>5.</td>
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<td>6. ( \angle ABC \cong \angle DEF )</td>
<td>6.</td>
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Now that we have proven theorems 1 & 2, we can use them in proofs.

Example #8

Given: \( \angle A \) is a right angle
\( \angle C \) is a right angle

Prove: \( \angle A \cong \angle C \)

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Example #9

Given: Diagram as shown

Prove: \( \angle EFG \cong \angle HFJ \)

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**Example #10**

Given:  
- \( m\angle 1 = 50 \)
- \( m\angle 2 = 40 \)
- \( \angle X \) is a right angle

Prove:  
- \( \angle ABC \cong \angle X \)

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![Diagram showing angles 1, 2, and X]
3.4 **Midpoints, Bisectors, & Perpendicularity**

- **Midpoints & Bisectors of Segments**
  - A point (or segment, ray, or line) that divides a segment into two congruent segments **bisects** the segment.
    - The bisection point is called the **midpoint** of the segment.
    - Only segments have midpoints.
      - Given: \( M \) is the midpoint of \( \overline{AB} \)
      - Conclusion: ______________________________________________________________________

- **Trisection Points & Trisecting a Segment**
  - Two points (or segments, rays, or lines) that divide a segment into three congruent segments **trisect** the segment.
    - The two points at which the segment is divided are called the **trisection points** of the segment.
    - Only segments have trisection points.
      - Given: \( R \) and \( S \) are trisection points of \( \overline{AC} \)
      - Conclusion: _______________________________________________________

- **Angle Bisectors**
  - A ray that divides an angle into two congruent angles **bisects** the angle.
    - The dividing ray is called the **bisector** of the angle.
      - Given: \( \overline{AW} \) bisects \( \angle TAO \)
      - Conclusion: _______________________________________________________

- **Angle Trisectors**
  - Two rays that divide an angle into three congruent angles **trisect** the angle.
    - The two dividing rays are called the **trisectors** of the angle.
      - Given: \( \overline{BH} \) and \( \overline{BR} \) trisect \( \angle TBE \)
      - Conclusion: _______________________________________________________

*Unit 3: Triangles*
Examples ~

1. Each figure shows a triangle with one of its angle bisectors.
   a) $\angle SUT = 34^\circ$. Find $\angle 1$.
   b) Find $m\angle SQR$ if $m\angle 2 = 13^\circ$.

2. The figure shows a triangle with one of its angle bisectors.
   Find $x$ if $m\angle 2 = 4x + 5$ and $m\angle 1 = 5x - 2$.

3. Given: $\overline{PS}$ bisects $\angle RPO$
   Prove: $\angle RPS \cong \angle OPS$

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4. Given: $\overline{CM}$ bisects $\overline{AB}$
   Prove: $\overline{AM} \cong \overline{MB}$

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Perpendicular Lines, Rays, & Segments

- Perpendicularly, right angles, & 90° measurements all go together.
- Lines, rays, or segments that intersect at right angles are perpendicular (\(\perp\)).
  - A pair of perpendicular lines forms four right angles.

Do not assume perpendicularity from a diagram!
- In the figure at the right, the mark inside the angle (\(\neg\)) indicates that \(\angle G\) is a right angle.
  - **Given:** \(\overline{GR} \perp \overline{GT}\)
  - **Conclusion:** ________________________________

Examples ~

5. Given: \(\overline{AB} \perp \overline{BD}\)
   \(\overline{DC} \perp \overline{AC}\)
   Prove: \(\angle B \cong \angle C\)

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6. Given: \(\overline{EH} \perp \overline{HG}\)
   Name all the angles you can **prove** to be right angles.
3.5 **Complementary & Supplementary Angles**

- **Complementary angles** — two angles whose sum is 90° (a right angle)
  - Each of the two angles is called the complement of the other.

- **Supplementary angles** — two angles whose sum is 180° (a straight angle)
  - Each of the two angles is called the supplement of the other.

- **Linear Pair Theorem** ~ If two angles form a linear pair, then they are supplementary.
  - If two angles are congruent and supplementary, then each is a right angle.

**Examples ~**

1. Given: \( \angle TVK \) is a right \( \angle \).
   
   Prove: \( \angle 1 \) is complementary to \( \angle 2 \).

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<tr>
<td>( \angle TVK ) is a right ( \angle )</td>
<td>Given</td>
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<tr>
<td>( \angle 1 ) is complementary to ( \angle 2 )</td>
<td>Definition of complementary angles</td>
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2. Given: Diagram as shown
   
   Prove: \( \angle 1 \) is supplementary to \( \angle 2 \).

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<td>Diagram as shown</td>
<td>Given</td>
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<td>( \angle 1 ) is supplementary to ( \angle 2 )</td>
<td>Definition of supplementary angles</td>
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Congruent Complements & Supplements

In the diagram below, \( \angle 1 \) is supplementary to \( \angle A \), and \( \angle 2 \) is also supplementary to \( \angle A \).

![Diagram](image)

- How large is \( \angle 1 \)? How large is \( \angle 2 \)? How does \( \angle 1 \) compare with \( \angle 2 \)?

**Theorem:** If angles are supplementary to the same angle, then they are congruent.

3. Given: \( \angle 3 \) is supp. to \( \angle 4 \)
   \( \angle 5 \) is supp. to \( \angle 4 \)

Prove: \( \angle 3 \cong \angle 5 \)

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<tr>
<td>1. ( \angle 3 ) is supp. to ( \angle 4 )</td>
<td>1.</td>
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<tr>
<td>2. ( m\angle 3 + m\angle 4 = 180 )</td>
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<td>3. ( m\angle 3 = 180 - m\angle 4 )</td>
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<td>4. ( \angle 5 ) is supp. to ( \angle 4 )</td>
<td>4.</td>
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<tr>
<td>5. ( m\angle 5 + m\angle 4 = 180 )</td>
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<tr>
<td>6. ( m\angle 5 = 180 - m\angle 4 )</td>
<td>6.</td>
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<tr>
<td>7. ( \angle 3 \cong \angle 5 )</td>
<td>7.</td>
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**Congruent Supplements Theorems:**
- If angles are supplementary to the same angle, then they are congruent.
- If angles are supplementary to congruent angles, then they are congruent.

**Congruent Complements Theorems:**
- If angles are complementary to the same angle, then they are congruent.
- If angles are complementary to congruent angles, then they are congruent.
4. Given: \( \angle 1 \) is supp. to \( \angle 2 \)
   \( \angle 3 \) is supp. to \( \angle 4 \)
   \( \angle 1 \cong \angle 4 \)
Prove: \( \angle 2 \cong \angle 3 \)

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5. Given: \( \angle A \) is comp. to \( \angle C \)
   \( \angle DBC \) is comp. to \( \angle C \)
Prove: _______________________________  

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6. Given: Diagram as shown
   Prove: \( \angle ABE \cong \angle DBC \)
   *Do not use vertical angles.*

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Unit 3: Triangles
7. Given: $\overrightarrow{EA} \perp \overrightarrow{EC}$ and $\overrightarrow{EB} \perp \overrightarrow{ED}$  
Prove: $\angle CED \cong \angle AEB$

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[Diagram of points A, B, C, D, and E with lines and angles labeled as per the problem.]
3.6 PROPERTIES OF SEGMENTS & ANGLES

The Addition Properties

Segment Addition Property ~ If a segment is added to two congruent segments, the sums are congruent.

If \( \overline{AB} \cong \overline{CD} \), does \( \overline{AC} \cong \overline{BD} \)? Explain.

Angle Addition Property ~ If an angle is added to two congruent angles, the sums are congruent.

Does a similar relationship hold for angles?

- Does \( m\angle EFH = m\angle JFG \)? Explain.

More Addition Properties

- If congruent segments are added to congruent segments, the sums are congruent.
- If congruent angles are added to congruent angles, the sums are congruent.

Using the Addition Properties Proofs:

- An addition property is used when the segments or angles in the conclusion are greater than those in the given information.

Reflexive Property: Any segment or angle is congruent to itself.

- Whenever a segment or an angle is shared by two figures, we can say that the segment or angle is congruent to itself.
The Subtraction Properties & Proofs

- A subtraction property is used when the segments or angles in the conclusion are *smaller than* those in the given information.

### Segment and Angle Subtraction Properties

- If a segment (or angle) is subtracted from congruent segments (or angles), the differences are congruent.
- If congruent segments (or angles) are subtracted from congruent segments (or angles), the differences are congruent.

1. Given: Diagram as shown.
   \[ AB \cong CD \]
   Explain how \( AC \cong BD \).

2. Given: Diagram as shown (w/tick marks).
   Explain how \( CA \cong CB \).

3. Given: Diagram as shown.
   \[ \angle SQU \cong \angle TQR \]
   Explain how \( \angle SQT \cong \angle UQR \).

4. Given: \( \overline{GJ} \cong HK \)
   Prove: \( \overline{GH} \cong JK \)

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*Unit 3: Triangles*
5. **Given:** \( \angle NOP \cong \angle NPO \)
   \( \angle ROP \cong \angle RPO \)
   **Prove:** \( \angle NOR \cong \angle NPR \)

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**Transitive Properties of Congruence**

- Suppose that \( \angle A \cong \angle B \) and \( \angle A \cong \angle C \). Is \( \angle B \cong \angle C \)?
  - If angles (or segments) are congruent to the same angle (or segment), they are congruent to each other.
  - If angles (or segments) are congruent to congruent angles (or segments), they are congruent to each other.

**Substitution Property**

- (Solving for a variable \( x \) & *substituting* the value found for that variable.)

6. **Given:** \( m\angle 1 + m\angle 2 = 90 \),
   \( \angle 1 \cong \angle 3 \)
   **Prove:** \( m\angle 3 + m\angle 2 = 90^\circ \)

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7. Given: \( \angle 1 \cong \angle 2 \)
\( \angle 1 \) is comp. to \( \angle 4 \)
\( RP \perp OP \)
\( \angle 4 \cong \angle 5 \)
Prove: \( \angle 2 \cong \angle 3 \)

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<td>( \angle 4 \cong \angle 5 )</td>
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<tr>
<td>( \angle 2 \cong \angle 3 )</td>
<td>( \angle 2 \cong \angle 3 )</td>
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3.7 **Angles Formed by Intersecting Lines**

- **Opposite rays** – two collinear rays that have a common endpoint & extend in different directions

**Vertical Angles**

- Whenever two lines intersect, two pairs of vertical angles are formed.
- Two angles are vertical angles if the rays forming the sides of one & the rays forming the sides of the other are opposite rays.

**Vertical Angles Theorem** ~ Vertical angles are congruent.

1. Given: \( \angle 4 \cong \angle 6 \)
   Prove: \( \angle 5 \cong \angle 6 \)

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<td>( \angle 4 \cong \angle 6 )</td>
<td>Given</td>
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<td>( \angle 5 )</td>
<td>?</td>
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<tr>
<td>( \angle 6 )</td>
<td>?</td>
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2. Given: \( \angle O \) is comp. to \( \angle 2 \)
   \( \angle J \) is comp. to \( \angle 1 \)
   Prove: \( \angle O \cong \angle J \)

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<td>( \angle O \cong \angle 2 )</td>
<td>Given</td>
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<tr>
<td>( \angle J \cong \angle 1 )</td>
<td>Given</td>
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<tr>
<td>( \angle O )</td>
<td>?</td>
</tr>
<tr>
<td>( \angle J )</td>
<td>?</td>
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Theorems Involving Parallel Lines

- The Parallel Postulate
  - Through a point not on a line there is exactly one parallel to the given line.

- Theorems on Parallel Lines, Transversals &/or Angles
  - If two parallel lines are cut by a transversal, then any pair of the angles formed are either congruent or supplementary.

- Angles formed from Parallel Lines
  - If two parallel lines are cut by a transversal...

### Alternate Exterior Angles
- Congruent: \( \angle 1 \cong \angle 8 \)

### Alternate Interior Angles
- Congruent: \( \angle 1 \cong \angle 2 \)

### Corresponding Angles
- Congruent: \( \angle 1 \cong \angle 5 \)

### Same-Side Interior Angles
- Supplementary: \( m\angle 3 + m\angle 5 = 180^\circ \)

### Same-Side Exterior Angles
- Supplementary: \( m\angle 1 + m\angle 7 = 180^\circ \)

- In a plane, if a line is perpendicular to one of two parallel lines, it is perpendicular to the other.
  - If \( a \parallel b \) and \( c \perp a \), then \( c \perp b \).

- If two lines are parallel to a third line, they are parallel to each other.
  - If \( a \parallel b \) and \( b \parallel c \), then \( a \parallel c \).
Theorems & Postulates Related to Parallel Lines

- **Corresponding Angles Postulate** ~ If a transversal intersects two parallel lines, then corresponding angles are congruent.

- **Converse of the Corresponding Angles Postulate** ~ If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.

- **Alternate Interior Angles Theorem** ~ If a transversal intersects two parallel lines, then alternate interior angles are congruent.

- **Same-Side Interior Angles Theorem** ~ If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

- **Converse of the Alternate Interior Angles Theorem** ~ If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.

- **Converse of the Same-side Interior Angles Theorem** ~ If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

3. Given: $a \parallel b$

   $\angle 1$ is supplementary to $\angle 3$

   Prove: $m \parallel p$

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<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<tr>
<td>$\angle 1$ is supplementary to $\angle 3$</td>
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<tr>
<td>$m \parallel p$</td>
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*Unit 3: Triangles*
4. Given: \( a \parallel b \)
\[ \angle 1 \cong \angle 2 \]
Prove: \( m \parallel p \)

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\( m \parallel p \)
### 3.8 Triangle Proofs

#### Theorems Involving Triangles
- **Triangle Angle-Sum Theorem**
  - The sum of the measures of the angles of a triangle is 180 degrees.

The diagram below shows that when you tear off the corners of any triangle, you can place the angles together to form a straight angle.

#### PROVE IT!

**Given:** \( \ell \parallel \overline{AC} \)

**Prove:** \( m\angle 1 + m\angle 2 + m\angle 3 = 180 \)

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<tbody>
<tr>
<td>( m\angle 1 + m\angle 2 + m\angle 3 = 180 )</td>
<td>Angle Sum Theorem</td>
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Triangle Exterior-Angle Theorem
- The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

**PROVE IT!**

Given: \( \triangle ABC \) with exterior angle \( \angle ACD \)
Prove: \( m\angle ACD = m\angle A + m\angle B \)

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Isosceles Triangle Theorems
- If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
  - Converse of Isosceles Triangle Theorem ~ If two angles of a triangle are congruent, then the sides opposite those angles are congruent.
- If a line bisects the vertex angle of an isosceles triangle, then the line is also the perpendicular bisector of the base.
1. Given: $\overline{AB} \cong \overline{BC}$
   Prove: $\angle 1 \cong \angle 3$

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Unit 3: Triangles